Probabilistic Verification of an Ant-Based Swarming Algorithm

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Abstract: Control algorithms for robot swarms are often inspired by decentralised problem/solving systems found in nature. In this paper we conduct a formal analysis of an algorithm inspired by the foraging behaviour of ants, where a swarm of flying vehicles searches for a target at some unknown location. We give the results of checking probabilistic temporal properties that complement simulation results, and would facilitate the logistics of swarm deployment.

1 Introduction

A robot swarm is comprised of some number of simple, homogeneous robots, working together to achieve objectives in some environment without centralised control [5]. Coordination between members of the swarm is achieved through self-organisation and local interactions.

Swarm behaviours are generally analysed through simulation and observations of real implementations. The formal analysis of swarm behaviours can complement the design of swarm algorithms by revealing potential problems that may go unnoticed by empirical analysis [1].

A common approach in swarm robotics has been to develop control algorithms based on abstractions of natural systems. In particular, much work has been conducted to develop control algorithms based on the behaviours of social insects. In [3] a swarm of Micro Air Vehicles (MAVs) attempts to form a communication pathway between multiple ground users in a disaster area. The control algorithm for each MAV is inspired by the stigmergic foraging behaviour of army ants which maintain pheromone paths between their nest and food sources.

We have applied probabilistic temporal verification to the scenario presented in [3], generating paramaterized formal models for the probabilistic model checker PRISM, which we use to either exhaustively or statistically test probabilistic reachability and reward-based properties. We demonstrate how values pertaining to the logistics of deployments of swarms of MAVs, that would be unobtainable through simulation alone, can be calculated by exhaustively checking reward-based properties in the models.

This extended abstract gives an overview of the scenario and presents the results of checking probabilistic temporal logic properties in the models.

2 Scenario

The scenario to which we apply probabilistic model checking is presented in [3]. Here, a simulated swarm of MAVs is deployed in order to establish a robust emergency communication network between a *target user*, situated at some unknown location, and the base station wherefrom the swarm is launched.

Figure 1 shows a Y-junction grid consisting of possible positions that MAVs will ideally adopt in their search for the target user. MAVs are launched at regular intervals from position $(0, 0)$ on the grid, and are in the *exploring* state. In the exploring state a MAV navigates through the grid. When a MAV reaches a position in the grid where there is no other MAV it will change to the *node* state and remain at that position, acting as a platform upon which other MAVs can "deposit" virtual pheromone. When a MAV in the exploring state reaches a position in the grid where there is already a MAV in the node state, it continues moving outward and makes a probabilistic choice on which branch to take, determined by the levels of pheromone deposited at the next positions on the left and right branches. Pheromone levels dissipate gradually over time and when they are depleted a MAV in the node state changes to the *returning* state. It then navigates back through the grid towards the base node similarly to a MAV in the exploring state.

3 Modelling

Models of the scenario were constructed using the *probabilistic model checker* PRISM [4]. Given a probabilistic model of a system, PRISM can be used to analyse both temporal and probabilistic properties of the input model by exhaustively checking some logical requirement against all possible behaviours. Properties to be checked can be specified using *probabilistic temporal logics* such as Probabilistic Computation Tree Logic (PCTL). PCTL consists of clas-

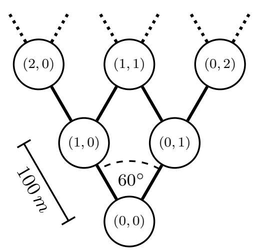

Figure 1: The Y-junction grid illustrating the ideal positions for MAVs.

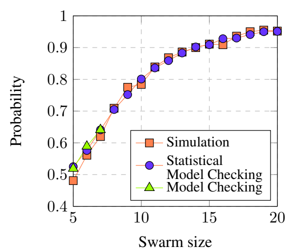

Figure 2: The mean probability of finding the user within 30 minutes.

sical logical operators, temporal operators,, and the probabilistic operator $\mathrm{P}_{\bowtie\gamma}(\phi)$ where $\bowtie \in \{<, \leq, >, \geq\}$ is a relational operator and γ is a probability threshold. PCTL can therefore be used to specify properties such as $\mathrm{P}_{\geq 0.5}(\Diamond\phi)$, meaning "$\phi$ holds at some future point with a probability of at least 0.5". PRISM allows properties to be expressed which evaluate to a numerical value, for instance $\mathrm{P}_{=?}(\Diamond\phi)$, "the probability of ϕ being true at some point in the future".

PRISM can be used to reason about other measurable aspects of model behaviours. Rewards can be associated with states and properties relating to expected values for these rewards can be checked in models. The R operator allows properties to be expressed such as the reachability reward property $\mathrm{R}_{=?}(\Diamond\phi)$, "what is the expected reward for reaching a state where ϕ is true".

In our models we consider only the moments in time where each MAV is exactly at some ideal position (i, j). Since MAVs are behaviourally identical we can use a counting abstraction to record the number of MAVs at each location. In addition we ignore the possibility of MAVs colliding or getting lost. We refer the reader to [2] for a comprehensive description of the model.

4 Experiments

To validate our model we applied statistical model checking using the PRISM discrete-event simulator and compared our results to those obtained from the simulations conducted in [3]. The mean probability of establishing contact with a user within 30 minutes was calculated over a series of 500 simulations for varying swarm sizes. The target user was located at some randomly determined location within a 60 degree arc in a known cardinal direction from the base node at a distance of $\approx 200/500\,m$.

Figure 2 compares the results of calculating the mean probability over all locations of a MAV establishing contact with the target user, to the results presented in [3]. Statistical model checking results were obtained using 500 discrete-event simulation samples with an average confidence interval of $\pm 2\%$ based on a 99.0% confidence level.

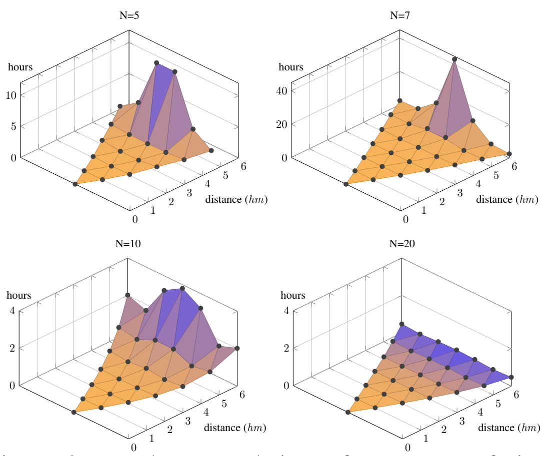

Figure 3: Total expected times for swarms of size N to establish communication with a user with probability 1.

By associating a reward of one with each state in our models we can calculate the total time expected for the swarm to establish contact with a user at (i, j) with probability 1. In Figure 3 we show the total expected time in hours for a deployment of N MAVs to establish communication with the target user with a probability 1. Results where $N > 7$ were obtained using statistical methods over 4000 samples.

5 Conclusions and Further Work

We have constructed formal probabilistic models making some simplifying assumptions, and clearly shown a close correspondence between these models and the simulations conducted in the original scenario. We have checked both probabilistic and reward-based properties in our model, where the resultant calculated values could be used to plan the deployment of a swarm of MAVs where establishing contact with a user must be guaranteed, or achieved with a probability that exceeds some given threshold.

We aim to further abstract our approach so that the techniques that we have developed here can be applied to a broader range of swarm algorithms where stigmergic communication is used to coordinate the behaviour of the swarm.

Acknowledgments

The authors would like to thank the Networks Sciences and Technology Initiative (NeST) of the University of Liverpool for the use of their computing facilities. The first author would like to acknowledge the funding received from the Sir Joseph Rotblat Alumni Scholarship.

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